Hide and Malin[1] confined their attention to displacements of magnetic pattern in longitude alone and observed a high degree of correlation between the two fields requiring a displacement of the magnetic pattern 160° to the east. I extend their studies here by considering displacements in latitude. This study will require transforming spherical harmonic series from one polar axis to another.

A rotation can be completely specified by the angle of rotation and the axis about which it takes place. An arbitrary rotation may be represented as a product of rotations. A set of all such rotations forms a group and is completely defined by a matrix. Consider a rotation about ox through an angle $\bar{\theta}$ and rotation about oz through angle $\bar{\varphi}$. Donating the matrices corresponding to each of these rotations by T_φ, T_θ, produces[4]

$$T_g = T_\varphi T_\theta \tag{1}$$

Rotation of spherical harmonic series about the axis ox and oz leads to[4]

$$T_g = e^{-im\bar{\varphi}}\, u_n^m(\bar{\theta}) \tag{2}$$

where

$$u_n^m(\bar{\theta}) = \frac{(-1)^{n-m}}{2^n(n-m)!}\sqrt{\frac{(n-m)!}{(n+m)!}}\,(1-\sin\theta)^{-m/2}$$

$$(1+\sin\theta)^{-m/2}\frac{d^n}{d(\sin\theta)^n}\left[(1-\sin\bar{\theta})^{n-m}(1+\sin\bar{\theta})^{n+m}\right] \tag{3}$$

where m denotes the order of the harmonic and n the degree, m and n being positive integers. Let a_n^m and b_n^m be spherical harmonic coefficients in the expansion of magnetic potential v. If the magnetic pattern is displaced longitudinally by angle $\bar{\varphi}$ and in latitude by angle $\bar{\theta}$ the coefficients will be modified as

$$a_n^m \to [a_n^m \cos m\bar{\varphi} - b_n^m \sin m\bar{\varphi}]\, u_n^m(\bar{\theta})$$
$$b_n^m \to [a_n^m \sin m\bar{\varphi} + b_n^m \cos m\bar{\varphi}]\, u_n^m(\bar{\theta}) \tag{4}$$

If A_n^m and B_n^m be the spherical harmonic coefficients in the spherical harmonic expansion of gravitational potential U, then the correlation coefficient k defined as

$$k = \frac{\sum_{m,n}(a_n^m A_n^m + b_n^m B_n^m)}{\left[\sum_{m,n}[(a_n^m)^2 + (b_n^m)^2]\right]^{1/2}\left[\sum_{m,n}[(A_n^m)^2 + (B_n^m)^2]\right]^{1/2}} \tag{5}$$

will remain unchanged on further displacement of magnetic pattern in latitude. The high degree of correlation observed by Hide and Malin[1] requiring a displacement of the magnetic pattern 160° to the east is not therefore affected by the displacement in latitude. This further supports Hide's suggestion[2] that core motions are probably chiefly zonal.

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Earthquake Lightning

In some parts of the world, earthquakes are often accompanied by ball lightning, stroke lightning and sheet lightning[1]. The only causal connexion that seems possible is that the seismic strains of the earthquake somehow cause an electric field in the air, which in turn produces ball lightning[2] and stroke and sheet lightning. What is the mechanism of this "seismoelectric effect"?

It is suggested by Terada[3] that the streaming potential of subterranean capillary flow of water causes these electric fields. We estimate, however, that the seismic stresses necessary to produce breakdown fields by this mechanism are several orders of magnitude greater than exist during earthquakes. Electrostatic generation by dust, which is probably important in volcano lightning, is not significant in these earthquakes.

We propose that the piezoelectric effect in the Earth's crust causes the electric field.

The only significant piezoelectric constituent of the crust seems to be quartz. The mere presence of quartz is not sufficient; there must be the right kind of long range crystalline order or texture, for example, $m3:m$ or ∞m. The existence and magnitude of just such order are known from piezoelectric prospecting for quartz-bearing ores[4-6]. The range of order relevant to the seismoelectric effect is the wavelength of seismic waves (~ 2 km). Natural geological structures of this size may exhibit effective piezoelectric coefficients of the order of several per cent that of x cut single crystal quartz.

The long range order implied by these measurements is probably the result of the stress history shared by rocks in one tectonic unit[7-9]. At the relevant temperatures the z axes of quartz crystals tend to line up along the principal direction (eigenvector) of greatest stress. In one rock, for example, 50 per cent of the z axes are within 6·4 degrees of the principal stress direction[10]. There are several processes which can then order the x axes of the z ordered quartz grains. Secondary stresses may order the flats of quartz grains by mechanical action, thus ordering the x axes up to sense. An ordering of their senses occurs in the elimination of Dauphiné twinning by a shift in the direction of principal stress[11].

In rock with a mean piezoelectric coefficient several per cent that of x cut single crystal quartz, and with typical seismic stress changes 30–300 bars, an earthquake makes an average electric field of 500–5,000 V cm^{-1}. For distances of the order of half the seismic wavelength, the generated voltage is 5×10^7 to 5×10^8 V, which is comparable with the voltage responsible for lightning in storms. The impedance presented to this generator by a thin stratum of conductive soil or by conduction through the rock itself does not significantly load it at typical seismic frequencies.

For example, the North Idu peninsula earthquake of November 26, 1930, the best documented instance[3,12] of seismoelectricity (over fifteen hundred sightings), occurred in a region with widespread quartz rich lava flows. The geology and petrology of this area have been extensively treated by Kuno[13-15]; near Mt Hakone, the approximate centre of earthquake lightning, most of the lava flows contain between 15 and 30 per cent by weight of free silica, usually in the form of quartz. Some rocks contain up to 43 per cent free quartz. The rocks are usually crystalline and rarely glassy. In addition to these lava flows there are many regions with exposed dikes and plugs which contain large amounts of quartz and which have crystallized very slowly. One particular quartz diorite plug north of Mt Hakone forms a whole mountain, Yagura-dâke, approximately 1 km across. This plug is only a few km north of the active Hakone fault along which there was extensive slippage during the earthquake of November 26, 1930. The whole North Idu region has been undergoing tectonic processes since the late Tertiary[16] with consequent strong regional shearing stresses. Thus we conclude that extensive long range ordering of quartz rich rocks has probably taken place in this region.

These calculations make certain predictions possible. We expect that field measurements will show ground voltage differences in the North Idu region during earthquakes, differences sometimes large enough to cause atmospheric electric discharges. It surprises us that while minute piezomagnetic fields of seismic origin have been explored[17], no attention seems to have been paid to such gross piezoelectric fields.

There should also be very low frequency electromagnetic radiation from seismoelectric waves ranging from 10 Hz, the approximate upper frequency of seismic waves, to well below 1 Hz, and from transient stress changes at higher frequencies. The seismic waves provide an effective 1 km^3 antenna carrying a current of some 1 to 10 A with a spectral maximum near 1·5 Hz, the approximate maximum for seismic waves. The radiation takes place into the atmospheric cavity whose fundamental frequency is approximately 7 Hz. Electromagnetic radiation from 1·5 Hz seismic waves will be of low power ($\ll$1 W) but radiation from higher frequency transients will be much more intense.

There will also be electrical precursors to earthquakes resulting from changes in stress near earthquake foci. There is a tradition in Japan of predicting earthquakes, sometimes with great saving of life, from unusual clear sky lightning[3]. It may be possible to put this kind of prediction on a more systematic basis using more sensitive and quantitative electric measuring instruments than earthquake lightning.

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Crack Growth under Creep Conditions

In recent years there has been a growing awareness that the techniques of linear elastic fracture mechanics provide a powerful method for quantifying the behaviour of engineering components containing crack-like defects. In the fast fracture of high strength, low toughness materials[1] and in fatigue crack growth[2], it is now established that data from laboratory tests can be confidently applied to predict the integrity of a structure or crack extension in service using sharp crack stress intensity factors. More recently, a study of fatigue crack initiation from sharp notches in mild steel[3] has shown that the number of cycles to initiate a crack can also be calculated using stress intensity factors. In components which operate at elevated temperatures there is a need to develop methods which describe the growth of crack-like defects in creed conditions. This is particularly relevant to welded joints where defects can be evident after stress relief and assessments are required of fitness for use.

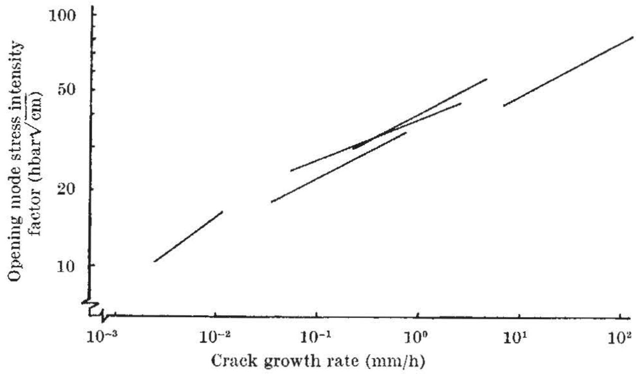

Fig. 1. The relationship between sharp crack stress intensity factor and crack growth rate for 2·25 per cent Cr, 1 per cent Mo steel under creep conditions at 565° C.

The mechanisms of creep fracture in terms of the nucleation and growth of cavities and cracking at grain boundary triple point junctions have been extensively studied[4-7]. Direct studies of crack growth in creep conditions leading to a quantitative description of material behaviour are, however, relatively few[8,9]. This has no doubt been principally because of the lack of a convenient technique for the measurement of crack length at elevated temperature. This difficulty can be overcome by using the electrical resistance method as described by Gilbey and Pearson[10]. The technique has been used in these laboratories during the past three years and has proved reliable; good discrimination has been achieved by the use of high specimen currents, high sensitivity recorders and by careful attention to experimental detail.

Work has been carried out on samples of 2·25 per cent Cr 1 per cent Mo steampipe steel, which had been quenched from 1,350° C to simulate the heat affected zone of a weld. Edge notched rectangular cross-section tensile specimens were tested in air at 565° C using a conventional lever loaded creep machine and the voltage drop across the crack was monitored continuously. The specimen dimensions were: parallel gauge length 75 mm, width 25 mm, thickness 5 mm, and initial notch depth 2·5 mm. Dead loads were applied which produced nominal stresses from 10 to 44 hbar and caused fracture in times up to about 1000 h. Some results are presented in Fig. 1 where each line represents a single test. An equation describing all the tests has the form

$$\dot{a} = \text{constant } K^n$$

where $\dot{a}$ is the crack growth rate, K is the stress intensity factor and where the value of n is about 5·5. This value is similar to the exponent that relates minimum creep rate to stress for materials of this type[11,12]. Correlations of growth rate with nominal applied stress or net section stress produced families of approximately parallel lines. There are obvious objections to applying linear elastic fracture mechanics to a creep situation, but we consider that the present findings represent a useful empirical relationship. Limited tests to verify the generality of the results, by testing at constant stress intensity factor by progressive load reduction with crack growth and by alteration of specimen dimensions, produced crack growth rates in agreement with the equation. The results encourage the view that it may be possible to establish relationships in high temperature crack propagation analogous to those which have already proved so useful in lower temperature applications.

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